Question	Scheme	Marks	AOs
1(a)	$u_2 = k-12,\ u_3 = k-\frac{24}{k-12}$	M1	1.1b
	$u_1+2u_2+u_3=0 \Rightarrow 2+2(k-12)+k-\frac{24}{k-12}=0$	dM1	1.1b
	$\Rightarrow 3k-22-\frac{24}{k-12}=0 \Rightarrow (3k-22)(k-12)-24=0$ $\Rightarrow 3k^2-36k-22k+264-24=0$ $\Rightarrow 3k^2-58k+240=0$ *	A1*	2.1
		(3)	
(b)	$k=6, \left(\frac{40}{3}\right)$	M1	1.1b
	k = 6 as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k-\frac{24}{2}$ o.e. but could be awarded for $u_3 = k-\frac{24}{\text{their } "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2+2"u_2"+"u_3"=0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2+2(k-12)+k-\frac{24}{k-12}=0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12)-24=0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of k = 6.

This may be awarded for any of

- $3k^2-58k+240=(ak\pm c)(bk\pm d)=0$ where $ab=3, cd=240$ followed by k =
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least k = 6

A1: Chooses k = 6 and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\frac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

Question	Scheme	Marks	AOs
2	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=\ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=\ldots$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\ldots\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=\ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3 marks)	

Notes

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be

seen as part of them writing down the sequence but must be the **first** term.
M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$
A1*: Correct proof

Alternative 1:
B1: Deduces the correct value for the sum to infinity (starting at n = 1) or the common ratio
M1: Calculates the required value by subtracting the first term from their sum to infinity
A1*: Correct proof

Alternative 2:
B1: Deduces the correct value of the **first** term or the common ratio.
M1: Splits the series into “odds” and “evens”, attempts the sum of both parts and calculates the required value by adding both sums
A1*: Correct proof

Alternative 3:
B1: Deduces the correct value of the **first** term
M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for “S”
A1*: Correct proof

Question	Scheme	Marks	AOs
3 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3} \frac{2}{x}\,dx$ or equivalent such as $2\int_{2.1}^{6.3} x^{-1}\,dx$ but must include the limits and the dx.

Condone $dx \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int \frac{1}{x}dx = \ln x$ and attempts to apply the limits (either way around)

Condone $\int \frac{2}{x}dx = p\ln x$ (including p = 1) or $\int \frac{2}{x}dx = p\ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{2}{x}dx = \ln x^2$, $\int \frac{2}{x}dx = 2\ln|x| + c$ and $\int \frac{2}{x}dx = 2\ln cx$ o.e. are also correct

$[p\ln x]_{2.1}^{6.3} = p\ln 6.3 - p\ln 2.1$ is sufficient evidence to award this mark

A1: CSO ln 9 . Also answer = $\ln 3^2$ so k = 9 is fine. Condone $\ln|9|$

The method mark must have been awarded. Do not accept answers such as $\ln\frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $[2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1 = 2.197 = \ln k \Rightarrow k = e^{2.197} = 8.998 = 9$

Question	Scheme	Marks	AOs
4(a)(i)	$a_1 = 3,\ a_2 = 5,\ a_3 = 3\ ...$	B1	1.1b
(ii)	2	B1	1.1b
		(2)	
(b)	$\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e.	M1	3.1a
	$= 339$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

(a)(i) Mark (a)(i) and (a)(ii) together.

B1: States the values of at least $a_2 = 5$ and $a_3 = 3$. This is sufficient but if more terms are given they must be correct. There is no need to see e.g. $a_2 = ...,\ a_3 = ...$ just look for values.

Allow an algebraic approach e.g. $a_{n+1} = 8 - a_n,\ a_{n+2} = 8 - (8 - a_n) = a_n$

A conclusion is **not** needed.

(a)(ii)

B1: States that the order of the periodic sequence is 2

Allow "second order", "it repeats every 2 numbers" or equivalent statements that convey the idea of the period being 2.

Note that ±2 is B0

(b)

M1: Attempts a **correct** method to find $\sum_{n=1}^{85} a_n$

For example $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$, $\sum_{n=1}^{85} a_n = \frac{84}{2}\times3+42\times5+3$ or $\sum_{n=1}^{85} a_n = 43\times(3+5)-5$

or $\sum_{n=1}^{85} a_n = 43\times3+42\times5$ or $\sum_{n=1}^{85} a_n = \frac{85}{2}\times8-1$

There may be other methods e.g. "Chunking": 5×(3 + 5) = 40, 40 × 8 = 320, 320 + 3×3 +2×5 = 339

A1: 339. Correct answer only scores both marks.

Attempts to use an AP formula score M0

Question	Scheme	Marks	AOs
5(a)(i)	e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40$ *	B1*	2.1
(ii)	$u_3=40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2\ (=28)$ **or** $u_4="28"+7\cos\left(\frac{3\pi}{2}\right)-5(-1)^3\ (=33)$	M1	1.1b
	$u_3=28$ and $u_4=33$	A1	1.1b
		(3)	
(b)(i)	$(u_5=)35$	B1	2.2a
(ii)	e.g. $\sum_{r=1}^{25} u_r = 6(35+40+"28"+"33")+35$	M1	3.1a
	$=851$	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

(i)

B1*: Correct application of the formula with n = 1 and proceeds correctly to achieve an answer of 40 with no errors. Note that e.g., $(u_2 =)35+7\cos\left(\frac{35\pi}{2}\right)-5(-1)^{35}=35+0+5=40$ scores B0

As a minimum need to see e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40$, $35+0+5=40$, $35+5=40$, $35-5(-1)^1=40$

(ii)

M1: A correct attempt to use the formula to find a value for u_3 or u_4

Look for n = 2 substituted correctly into the given formula with $u_2=40$. May be implied by $u_3=28$

Or their calculated value of u_3 used with n = 3 substituted correctly into the given formula to find u_4

Condone use of calculator in degree mode which gives $u_3=41.989...$ which may imply this mark if no working is shown. If there is **no** working and u_3 is incorrect and u_4 is correct score M0A0

A1: Both correct $u_3=28$ and $u_4=33$ If 28, 33 are listed then allow M1A1.

For both correct values only score M1A1

(b)(i)

B1: $(u_5=)35$

(ii)

M1: Attempts a **<u>correct</u>** method to find $\sum_{r=1}^{25} u_r$

There are various ways e.g. attempts to add 35 to $6\times$the sum of their four values.

Some other examples are:

$\sum_{r=1}^{25} u_r = 7\times35+6\times40+6\times"28"+6\times"33"$, $\sum_{r=1}^{25} u_r = 7(35+40+"28"+"33")-(40+"28"+"33")$,

$\sum_{r=1}^{25} u_r = \frac{25}{4}(35+40+"28"+"33")+1$, $2(35+40+"28"+"33")=272$, $272\times3=816$, $816+35$

There may be other methods seen but the calculation must be correct for their values.

If there is no working, with incorrect u_3 and/or u_4 you will need to check if their answer implies a correct method using $6(35+40+"28"+"33")+35$

Attempts to use an AP/GP formula score M0

A1: 851 (Correct answer with no working scores both marks)